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Off-diagonal helicity density matrix elements for vector mesons produced in polarized e^+e^- processes

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Abstract. Final-state $q\bar{q}$ interactions give origin to nonzero values of the off-diagonal element $\rho_{1,-1}$ of the helicity density matrix of vector mesons produced in e^+e^- annihilations, as has been confirmed by recent OPAL data on ϕ, D^*, and K^*. New predictions are given for $\rho_{1,-1}$ of several mesons produced at large $x_{\rm E}$ and small $p_{\rm T}$ – i.e., collinear with the parent jet – in the annihilation of polarized e^+ and e^-; the results depend strongly on the elementary dynamics and allow further nontrivial tests of the standard model.

1 Introduction

In a series of papers [1]–[3], it has been pointed out how the final-state interactions between the q and $\bar{q}$ produced in e^+e^- annihilations – usually neglected, but indeed necessary – might give origin to nonzero values of spin observables which would otherwise be forced to vanish. The off-diagonal spin density matrix element $\rho_{1,-1}(V)$ of vector mesons may be sizeably different from zero [1,2] because of a coherent fragmentation process which takes into account $q\bar{q}$ interactions; indeed, predictions were given [3] for several spin-1 particles produced at LEP in two-jet events, provided that they carry a large fraction $x_{\rm E}$ of the parent quark energy and have a small intrinsic $p_{\rm T}$, i.e., that they are collinear with the parent jet.

The values of $\rho_{1,-1}(V)$ are related to the values of the off-diagonal helicity density matrix element $\rho_{+-;-+}(q\bar{q})$ of the $q\bar{q}$ pair, generated in the $e^-e^+ \to q\bar{q}$ process [3]:

$$\rho_{1,-1}(V) \simeq [1-\rho_{0,0}(V)]\,\rho_{+-;-+}(q\bar{q}) \tag{1}$$

where the value of the diagonal element $\rho_{0,0}(V)$ can be taken from data. The values of $\rho_{+-;-+}(q\bar{q})$ depend on the elementary short-distance dynamics and can be computed in the standard model. Thus, a measurement of $\rho_{1,-1}(V)$ is a further test of the constituent dynamics and is more significant than the usual measurement of cross sections in that it depends on the product of different elementary amplitudes rather than on squared moduli. With unpolarized e^+ and e^-,

$$\rho_{+-;-+}(q\bar{q}) = \frac{1}{4N_{q\bar{q}}} \sum_{\lambda_-,\lambda_+} M_{+-;\lambda_-\lambda_+}\, M^*_{-+;\lambda_-\lambda_+}\,, \tag{2}$$

where the M are the helicity amplitudes for the $e^-e^+ \to q\bar{q}$ process, and

$$4N_{q\bar{q}} = \sum_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+} |M_{\lambda_q\lambda_{\bar{q}};\lambda_-\lambda_+}|^2\,. \tag{3}$$

At LEP energy, $\sqrt{s} = M_Z$, one has [3]

$$\rho_{+-;-+}(q\bar{q}) \simeq \rho^{Z}_{+-;-+}(q\bar{q}) \simeq \frac{1}{2}\,\frac{(g_V^2-g_A^2)_q}{(g_V^2+g_A^2)_q}\,\frac{\sin^2\theta}{1+\cos^2\theta}\,. \tag{4}$$

where g_V and g_A are the standard model coupling constants [reported for convenience in (15)] and θ is the vector meson production angle in the e^-e^+ c.m. frame.

At lower energies, where weak interactions can be neglected, one has:

$$\rho_{+-;-+}(q\bar{q}) \simeq \rho^{\gamma}_{+-;-+}(q\bar{q}) = \frac{1}{2}\,\frac{\sin^2\theta}{1+\cos^2\theta}\,. \tag{5}$$

Equation (1) is in good agreement with OPAL Collaboration data on ϕ, D^*, and K^*, including the θ dependence induced by (4) [4,5]; however, no sizeable value of $\rho_{1,-1}(V)$ for $V=\rho,\phi$, and K^* was observed by the DELPHI Collaboration [6]. Further tests are thus necessary. Predictions for $\rho_{1,-1}(V)$, with $V=\phi, D^*$, or B^* produced in $NN \to VX$, $\gamma N \to VX$ and $\ell N \to \ell VX$ processes were given in [7].

We consider here again the process $e^+e^- \to VX$, assuming all possible polarization states for the initial leptons. This might not be a realistic case – polarized e^+e^- beams might not be available in the near future – but, as we shall see, the results show such a strong interesting dependence on the spin elementary dynamics that such a possibility should not be forgotten when future e^+e^- colliders are planned. Also, this work is the natural expansion and completion – with all possible cases and theoretical predictions taken into account – of the study undertaken in [3].

In the next section, we compute the value of $\rho_{+-;-+}(q\bar{q})$ with the most general spin states of e^+ and e^-; in Sect. 3, we obtain numerical estimates in several particular cases, and in Sect. 4, we give some comments and conclusions.

2 Computation of $\rho_{+-;-+}(q\bar{q})$

In the case of polarized initial leptons, (2) changes to:

$$\rho^{\rm pol}_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q}) = \frac{1}{N^{\rm pol}_{q\bar{q}}} \sum_{\lambda_-,\lambda_+,\lambda'_-,\lambda'_+} M_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+}\, \rho_{\lambda_-,\lambda_+;\lambda'_-,\lambda'_+}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};\lambda'_-,\lambda'_+} \tag{6}$$

with

$$N^{\rm pol}_{q\bar{q}} = \sum_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+,\lambda'_-,\lambda'_+} M_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+}\, \rho_{\lambda_-,\lambda_+;\lambda'_-,\lambda'_+}\, M^*_{\lambda_q,\lambda_{\bar{q}};\lambda'_-,\lambda'_+} \tag{7}$$

and where

$$\rho_{\lambda_-,\lambda_+;\lambda'_-,\lambda'_+}(e^-e^+) = \rho_{\lambda_-,\lambda'_-}(e^-)\, \rho_{\lambda_+,\lambda'_+}(e^+) \tag{8}$$

is the helicity density matrix of the incoming independent leptons.

The most general helicity density matrices for the incoming e^- and e^+ are given by

$$\rho(e^-) = \frac{1}{2}\begin{pmatrix} 1+\cos\alpha_- & {\rm e}^{-{\rm i}\beta_-}\sin\alpha_- \\ {\rm e}^{{\rm i}\beta_-}\sin\alpha_- & 1-\cos\alpha_- \end{pmatrix} \tag{9}$$

and

$$\rho(e^+) = \frac{1}{2}\begin{pmatrix} 1-\cos\alpha_+ & {\rm e}^{{\rm i}\beta_+}\sin\alpha_+ \\ {\rm e}^{-{\rm i}\beta_+}\sin\alpha_+ & 1+\cos\alpha_+ \end{pmatrix} \tag{10}$$

where α_- and β_- (α_+ and β_+) are respectively the polar and azimuthal angle of the e^- (e^+) spin vectors; we have chosen xz as the scattering plane with e^- (e^+) moving along the positive (negative) direction of z axis.

Insertion of (8)–(10) into (7) and (8), with lepton masses neglected, yields

$$\begin{aligned}\rho^{\rm pol}_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q}) = \frac{1}{4N^{\rm pol}_{q\bar{q}}}\Big[&(1+\cos\alpha_-)(1+\cos\alpha_+) \\ &\times M_{\lambda_q,\lambda_{\bar{q}};+,-}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};+,-} \\ &+ {\rm e}^{-{\rm i}(\beta_-+\beta_+)}(\sin\alpha_-\sin\alpha_+) \\ &\times M_{\lambda_q,\lambda_{\bar{q}};+,-}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};-,+} \\ &+ e^{i(\beta_-+\beta_+)}(\sin\alpha_-\sin\alpha_+) \\ &\times M_{\lambda_q,\lambda_{\bar{q}};-,+}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};+,-} \\ &+ (1-\cos\alpha_-)(1-\cos\alpha_+) \\ &\times M_{\lambda_q,\lambda_{\bar{q}};-,+}\, M^*_{\lambda'_q,\lambda'_{\bar{q}};-,+}\Big]\end{aligned} \tag{11}$$

with

$$\begin{aligned}4N^{\rm pol}_{q\bar{q}} = &(1+\cos\alpha_-)(1+\cos\alpha_+) \\ &\times \left[\,|M_{+-;+-}|^2 + |M_{-+;+-}|^2\,\right] \\ &+ (1-\cos\alpha_-)(1-\cos\alpha_+) \\ &\times \left[\,|M_{+-;-+}|^2 + |M_{-+;-+}|^2\,\right] \\ &+ 2\sin\alpha_-\sin\alpha_+ \\ &\times {\rm Re}\Big[{\rm e}^{-{\rm i}(\beta_-+\beta_+)}\big(M_{+-;+-}\, M^*_{+-;-+} \\ &+\; M_{-+;+-}\, M^*_{-+;-+}\big)\Big]\,.\end{aligned} \tag{12}$$

In the last equation, quark masses, compared to their energies, have been neglected.

The explicit expressions of the relevant $e^+e^- \to q\bar{q}$ c.m. helicity amplitudes are given by [3]:

$$\begin{aligned}M_{\pm\mp;\pm\mp} = {\rm e}^2(1+\cos\theta)\,[&e_q - g_Z(s)(g_V \mp g_A)_l \\ &\times (g_V \mp g_A)_q]\end{aligned} \tag{13}$$

$$\begin{aligned}M_{\pm\mp;\mp\pm} = {\rm e}^2(1-\cos\theta)\,[&e_q - g_Z(s)(g_V \pm g_A)_l \\ &\times (g_V \mp g_A)_q]\end{aligned} \tag{14}$$

with the usual standard model coupling constants:

$$\begin{aligned} g^l_V &= -\frac{1}{2} + 2\sin^2\theta_W & g^l_A &= -\frac{1}{2} \\ g^{u,c,t}_V &= \frac{1}{2} - \frac{4}{3}\sin^2\theta_W & g^{u,c,t}_A &= \frac{1}{2} \\ g^{d,s,b}_V &= -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W & g^{d,s,b}_A &= -\frac{1}{2} \\ g_Z(s) &= \frac{1}{4\sin^2\theta_W\cos^2\theta_W}\ \frac{s}{(s-M_Z^2)+{\rm i}M_Z\Gamma_Z}\,. \end{aligned} \tag{15}$$

By inserting (13) and (14) into (11), and (12) one obtains:

$$\begin{aligned}\rho^{\rm pol}_{+-;+-}(q\bar{q}) = \frac{1}{4N^{\rm pol}_{q\bar{q}}}\Big[&(1+\cos^2\theta)\ F^{\rm pol}_{1,q} + \cos\theta\ F^{\rm pol}_{2,q} \\ &+\sin^2\theta\ F^{\rm pol}_{3,q}\Big]\,,\end{aligned} \tag{16}$$

$$\begin{aligned}\rho^{\rm pol}_{+-;-+}(q\bar{q}) = \frac{1}{4N^{\rm pol}_{q\bar{q}}}\Big[&(1+\cos^2\theta)\,(F^{\rm pol}_{4,q} + {\rm i}F^{\rm pol}_{5,q}) \\ &+\cos\theta\,(F^{\rm pol}_{6,q} + {\rm i}F^{\rm pol}_{7,q}) \\ &+\sin^2\theta\,(F^{\rm pol}_{8,q} + {\rm i}F^{\rm pol}_{9,q})\Big],\end{aligned} \tag{17}$$

with

$$N^{\rm pol}_{q\bar{q}} = (1+\cos^2\theta)\,F^{\rm pol}_{10,q} + \cos\theta\,F^{\rm pol}_{11,q} + \sin^2\theta\,F^{\rm pol}_{12,q}\,. \tag{18}$$

The twelve functions $F^{\rm pol}_{i,q}$ depend on the spin directions of the incoming leptons:

$$F^{\rm pol}_{1,q} = (1+\cos\alpha_+)(1+\cos\alpha_-)$$

$$
\begin{aligned}
&\times\Big[e_q^2+|g_Z|^2\,(g_V-g_A)_l^2\,(g_V-g_A)_q^2\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V-g_A)_l\,(g_V-g_A)_q\Big]\\
&+\,(1-\cos\alpha_+)\,(1-\cos\alpha_-)\\
&\times\Big[e_q^2+|g_Z|^2\,(g_V+g_A)_l^2\,(g_V-g_A)_q^2\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V+g_A)_l\,(g_V-g_A)_q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{2,q}&=(1+\cos\alpha_+)\,(1+\cos\alpha_-)\,2\\
&\times\Big[e_q^2+|g_Z|^2\,(g_V-g_A)_l^2\,(g_V-g_A)_q^2\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V-g_A)_l\,(g_V-g_A)_q\Big]\\
&-\,(1-\cos\alpha_+)\,(1-\cos\alpha_-)\,2\\
&\times\Big[e_q^2+|g_Z|^2\,(g_V+g_A)_l^2\,(g_V-g_A)_q^2\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V+g_A)_l\,(g_V-g_A)_q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{3,q}&=2\,\sin\alpha_+\,\sin\alpha_-\Big[\cos(\beta_++\beta_-)\\
&\times\Big(e_q^2+|g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V-g_A)_q^2\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,(g_V-g_A)_q\Big)\\
&+\sin(\beta_++\beta_-)\,e_q\,2\,(\mathrm{Im}\,g_Z)\,g_A^l\,(g_V-g_A)_q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{4,q}&=2\,\sin\alpha_+\,\sin\alpha_-\Big[\cos(\beta_++\beta_-)\\
&\times[e_q^2+|g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V^2-g_A^2)_q\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q]\\
&+\sin(\beta_++\beta_-)\,e_q\,2\,(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{5,q}&=2\,\sin\alpha_+\,\sin\alpha_-\Big[\cos(\beta_++\beta_-)\,e_q\,2(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q\\
&+\,\sin(\beta_++\beta_-)\,e_q\,2\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{6,q}&=4\,\sin\alpha_+\,\sin\alpha_-\Big[-\cos(\beta_++\beta_-)\,e_q\,2\,(\mathrm{Re}\,g_Z)\,g_A^l\,g_A^q\\
&+\,\sin(\beta_++\beta_-)\,e_q\,2\,(\mathrm{Im}\,g_Z)\,g_V^l\,g_A^q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{7,q}&=4\,\sin\alpha_+\,\sin\alpha_-\Big[\cos(\beta_++\beta_-)\,e_q\,2\,(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q\\
&-\sin(\beta_++\beta_-)[e_q^2+|g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V^2-g_A^2)_q\\
&-e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q]\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{8,q}&=(1+\cos\alpha_+)\,(1+\cos\alpha_-)\\
&\times\Big[e_q^2+|g_Z|^2\,(g_V-g_A)_l^2\,(g_V^2-g_A^2)_q\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V-g_A)_l\,g_V^q\Big]\\
&+\,(1-\cos\alpha_+)\,(1-\cos\alpha_-)\\
&\times\Big[e_q^2+|g_Z|^2\,(g_V+g_A)_l^2\,(g_V^2-g_A^2)_q\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V+g_A)_l\,g_V^q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{9,q}&=(1+\cos\alpha_+)\,(1+\cos\alpha_-)\\
&\times\Big[e_q\,2\,(\mathrm{Im}\,g_Z)\,(g_V-g_A)_l\,g_A^q\Big]\\
&+\,(1-\cos\alpha_+)\,(1-\cos\alpha_-)\\
&\times\Big[e_q\,2\,(\mathrm{Im}\,g_Z)\,(g_V+g_A)_l\,g_A^q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{10,q}&=(1+\cos\alpha_+)\,(1+\cos\alpha_-)(1/2)\\
&\times\Big[e_q^2+|g_Z|^2\,(g_V-g_A)_l^2\,(g_V^2+g_A^2)_q\\
&-\,e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V-g_A)_l\,g_V^q\Big]\\
&+(1-\cos\alpha_+)\,(1-\cos\alpha_-)(1/2)\\
&\times\ \Big[e_q^2+|g_Z|^2\,(g_V+g_A)_l^2\,(g_V^2+g_A^2)_q\\
&-e_q\,2\,(\mathrm{Re}\,g_Z)\,(g_V+g_A)_l\,g_V^q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{11,q}&=(1+\cos\alpha_+)\,(1+\cos\alpha_-)\\
&\times2\,\Big[e_q\,(\mathrm{Re}\,g_Z)\,(g_V-g_A)_l\,g_A^q\\
&-\,|g_Z|^2\,(g_V-g_A)_l^2\,(g_Vg_A)_q\Big]\\
&-\,(1-\cos\alpha_+)\,(1-\cos\alpha_-)\\
&\times2\,\Big[e_q\,(\mathrm{Re}\,g_Z)\,(g_V+g_A)_l\,g_A^q\\
&-\,|g_Z|^2\,(g_V+g_A)_l^2\,(g_Vg_A)_q\Big]
\end{aligned}
$$

$$
\begin{aligned}
F^{\rm pol}_{12,q}&=(\sin\alpha_+\,\sin\alpha_-)\\
&\times\Big[\cos(\beta_++\beta_-)\,[e_q^2-e_q\,2\,(\mathrm{Re}\,g_Z)\,g_V^l\,g_V^q\\
&+\,|g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V^2+g_A^2)_q]+\sin(\beta_++\beta_-)\\
&\times[e_q\,2\,(\mathrm{Im}\,g_Z)\,g_A^l\,g_V^q]\Big]. \qquad (19)
\end{aligned}
$$

Equations (17)–(19) give, at lowest perturbative order in the standard model, the most general expression of $\rho^{\rm pol}_{+-;-+}(q\bar q)$ for a $q\bar q$ pair obtained in the annihilation process of polarized leptons, $e^-e^+\to q\bar q$; both weak and electromagnetic interactions (γ and Z_0 exchanges) are taken into account.

3 Numerical values of $\rho_{+-;-+}(q\bar q)$

Let us now consider different polarization states of e^- and e^+. We choose as possible spin directions the 3 coordinate axes, $\hat x$, $\hat y$, $\hat z$, with spin component $\pm1/2$ along these directions. The corresponding values of (α,β) in (9) and (10) are as follows:

$$
\begin{array}{lll}
+\hat x=(\pi/2,\,0) & +\hat y=(\pi/2,\,\pi/2) & +\hat z=(0,\,0)\\
-\hat x=(\pi/2,\,\pi) & -\hat y=(\pi/2,\,3\pi/2) & -\hat z=(\pi,\,\pi)
\end{array} \qquad (20)
$$

We then have a total of $6\times6=36$ possible initial spin states. Many of them will lead to the same value of $\rho^{\rm pol}_{+-;-+}(q\bar q)$, and it is convenient to group them into the following nine cases (notice that Case 3 is just listed for completeness, but it gives identically null results due to helicity conservation in the $e^-e^+Z_0$ and $e^-e^+\gamma$ vertices):

Case 1:

$\{P(e^-,+\hat{z})\,,\,P(e^+,+\hat{z})\};$

Case 2:

$\{P(e^-,+\hat{z})\,,\,P(e^+,+\hat{x})\},\ \{P(e^-,+\hat{z})\,,\,P(e^+,-\hat{x})\},$
$\{P(e^-,+\hat{z})\,,\,P(e^+,+\hat{y})\},\ \{P(e^-,+\hat{z})\,,\,P(e^+,-\hat{y})\},$
$\{P(e^-,+\hat{x})\,,\,P(e^+,+\hat{z})\},\ \{P(e^-,-\hat{x})\,,\,P(e^+,+\hat{z})\},$
$\{P(e^-,+\hat{y})\,,\,P(e^+,+\hat{z})\},\ \{P(e^-,-\hat{y})\,,\,P(e^+,+\hat{z})\};$

Case 3:

$\{P(e^-,+\hat{z})\,,\,P(e^+,-\hat{z})\},\ \{P(e^-,-\hat{z})\,,\,P(e^+,+\hat{z})\};$

Case 4:

$\{P(e^-,-\hat{z})\,,\,P(e^+,-\hat{z})\};$

Case 5:

$\{P(e^-,+\hat{x})\,,\,P(e^+,+\hat{x})\},\ \{P(e^-,-\hat{x})\,,\,P(e^+,-\hat{x})\},$
$\{P(e^-,+\hat{y})\,,\,P(e^+,-\hat{y})\},\ \{P(e^-,-\hat{y})\,,\,P(e^+,+\hat{y})\};$

Case 6:

$\{P(e^-,+\hat{x})\,,\,P(e^+,-\hat{x})\},\ \{P(e^-,-\hat{x})\,,\,P(e^+,+\hat{x})\},$
$\{P(e^-,+\hat{y})\,,\,P(e^+,+\hat{y})\},\ \{P(e^-,-\hat{y})\,,\,P(e^+,-\hat{y})\};$

Case 7:

$\{P(e^-,+\hat{x})\,,\,P(e^+,+\hat{y})\},\ \{P(e^-,+\hat{y})\,,\,P(e^+,+\hat{x})\},$
$\{P(e^-,-\hat{x})\,,\,P(e^+,-\hat{y})\},\ \{P(e^-,-\hat{y})\,,\,P(e^+,-\hat{x})\};$

Case 8:

$\{P(e^-,+\hat{x})\,,\,P(e^+,-\hat{y})\},\ \{P(e^-,-\hat{y})\,,\,P(e^+,+\hat{x})\},$
$\{P(e^-,-\hat{x})\,,\,P(e^+,+\hat{y})\},\ \{P(e^-,+\hat{y})\,,\,P(e^+,-\hat{x})\};$

Case 9:

$\{P(e^-,-\hat{z})\,,\,P(e^+,+\hat{x})\},\ \{P(e^-,-\hat{z})\,,\,P(e^+,+\hat{y})\},$
$\{P(e^-,-\hat{z})\,,\,P(e^+,-\hat{x})\},\ \{P(e^-,-\hat{z})\,,\,P(e^+,-\hat{y})\},$
$\{P(e^-,+\hat{x})\,,\,P(e^+,-\hat{z})\},\ \{P(e^-,-\hat{x})\,,\,P(e^+,-\hat{z})\},$
$\{P(e^-,+\hat{y})\,,\,P(e^+,-\hat{z})\},\ \{P(e^-,-\hat{y})\,,\,P(e^+,-\hat{z})\}.$

The corresponding expressions of the functions $F^{\rm pol}_{i,q}$ are given by:

Case 1:

$$\begin{aligned}
F^{\rm pol,C1}_{1,q} &= 4\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)^2_l\,(g_V - g_A)^2_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,(g_V - g_A)_l\,(g_V - g_A)_q\Big] \\
F^{\rm pol,C1}_{2,q} &= 2\,F^{\rm pol,C1}_{1,q} \\
F^{\rm pol,C1}_{3,q} &= F^{\rm pol,C1}_{4,q} = F^{\rm pol,C1}_{5,q} = F^{\rm pol,C1}_{6,q} = F^{\rm pol,C1}_{7,q} = F^{\rm pol,C1}_{12,q} = 0 \\
F^{\rm pol,C1}_{8,q} &= 4\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)^2_l\,(g_V^2 - g_A^2)_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,(g_V - g_A)_l\,g_V^q\Big] \\
F^{\rm pol,C1}_{9,q} &= e_q\,8\,({\rm Im}\,g_Z)\,(g_V - g_A)_l\,g_A^q \\
F^{\rm pol,C1}_{10,q} &= 2\,\Big[e_q^2 + |g_Z|^2\,(g_V - g_A)^2_l\,(g_V^2 + g_A^2)_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,(g_V - g_A)_l\,g_V^q\Big] \\
F^{\rm pol,C1}_{11,q} &= 8\Big[e_q\,({\rm Re}\,g_Z)\,(g_V - g_A)_l\,g_A^q \\
&\qquad - |g_Z|^2\,(g_V - g_A)^2_l\,(g_V\,g_A)_q\Big]\ ;
\end{aligned} \tag{21}$$

Case 2:

$$F^{\rm pol,C2}_{i,q} = (1/2)\,F^{\rm pol,C1}_{i,q} \quad (i = 1\text{–}12)\ ; \tag{22}$$

Case 3:

$$F^{\rm pol,C3}_{i,q} = 0 \quad (i = 1\text{–}12)\ ; \tag{23}$$

Case 4:

$$\begin{aligned}
F^{\rm pol,C4}_{1,q} &= 4\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)^2_l\,(g_V - g_A)^2_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,(g_V + g_A)_l\,(g_V - g_A)_q\Big] \\
F^{\rm pol,C4}_{2,q} &= -8\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)^2_l\,(g_V - g_A)^2_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,(g_V + g_A)_l\,(g_V - g_A)_q\Big] \\
F^{\rm pol,C4}_{3,q} &= F^{\rm pol,C4}_{4,q} = F^{\rm pol,C4}_{5,q} = F^{\rm pol,C4}_{6,q} = F^{\rm pol,C4}_{7,q} = F^{\rm pol,C4}_{12,q} = 0 \\
F^{\rm pol,C4}_{8,q} &= 4\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)^2_l\,(g_V^2 - g_A^2)_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,(g_V + g_A)_l\,g_V^q\Big] \\
F^{\rm pol,C4}_{9,q} &= e_q\,8\,({\rm Im}\,g_Z)\,(g_V + g_A)_l\,g_A^q \\
F^{\rm pol,C4}_{10,q} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V + g_A)^2_l\,(g_V^2 + g_A^2)_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,(g_V + g_A)_l\,g_V^q\Big] \\
F^{\rm pol,C4}_{11,q} &= 8\Big[- e_q\,({\rm Re}\,g_Z)\,(g_V + g_A)_l\,g_A^q \\
&\qquad + |g_Z|^2\,(g_V + g_A)^2_l\,(g_V g_A)_q\Big]\ ;
\end{aligned} \tag{24}$$

Case 5:

$$\begin{aligned}
F^{\rm pol,C5}_{1,q} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 + g_A^2)_l\,(g_V - g_A)^2_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,g_V^l\,(g_V - g_A)_q\Big] \\
F^{\rm pol,C5}_{2,q} &= 8\Big[- |g_Z|^2\,(g_V g_A)_l\,(g_V - g_A)^2_q \\
&\qquad + e_q\,({\rm Re}\,g_Z)\,g_A^l\,(g_V - g_A)_q\Big] \\
F^{\rm pol,C5}_{3,q} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V - g_A)^2_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,g_V^l\,(g_V - g_A)_q\Big] \\
F^{\rm pol,C5}_{4,q} &= 2\Big[e_q^2 + |g_Z|^2\,(g_V^2 - g_A^2)_l\,(g_V^2 - g_A^2)_q \\
&\qquad - e_q\,2\,({\rm Re}\,g_Z)\,g_V^l\,g_V^q\Big] \\
F^{\rm pol,C5}_{5,q} &= e_q\,4\,({\rm Im}\,g_Z)\,g_V^l\,g_A^q \\
F^{\rm pol,C5}_{6,q} &= -e_q\,8\,({\rm Re}\,g_Z)\,g_A^l\,g_A^q
\end{aligned}$$

$$F^{\mathrm{pol,C5}}_{7,q} = e_q\,8(\mathrm{Im}\,g_Z)\,g^l_A\,g^q_V$$
$$F^{\mathrm{pol,C5}}_{8,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V^2-g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\Big]$$
$$F^{\mathrm{pol,C5}}_{9,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_V\,g^q_A$$
$$F^{\mathrm{pol,C5}}_{10,q} = e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V^2+g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V$$
$$F^{\mathrm{pol,C5}}_{11,q} = 4\Big[-e_q(\mathrm{Re}\,g_Z)\,g^l_A g^q_A + |g_Z|^2\,2(g_V g_A)_l\,(g_V g_A)_q\Big]$$
$$F^{\mathrm{pol,C5}}_{12,q} = e_q^2 + |g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V^2+g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\ ; \tag{25}$$

Case 6:

$$F^{\mathrm{pol,C6}}_{1,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V-g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,(g_V-g_A)_q\Big]$$
$$F^{\mathrm{pol,C6}}_{2,q} = 8\Big[-|g_Z|^2\,(g_V g_A)_l\,(g_V-g_A)_q^2 + e_q\,(\mathrm{Re}\,g_Z)\,g^l_A\,(g_V-g_A)_q\Big]$$
$$F^{\mathrm{pol,C6}}_{3,q} = -2\Big[e_q^2 + |g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V-g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,(g_V-g_A)_q\Big]$$
$$F^{\mathrm{pol,C6}}_{4,q} = -2\Big[e_q^2 + |g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V^2-g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\Big]$$
$$F^{\mathrm{pol,C6}}_{5,q} = -e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_V\,g^q_A$$
$$F^{\mathrm{pol,C6}}_{6,q} = e_q\,8\,(\mathrm{Re}\,g_Z)\,g^l_A\,g^q_A$$
$$F^{\mathrm{pol,C6}}_{7,q} = -e_q\,8\,(\mathrm{Im}\,g_Z)\,g^l_A\,g^q_V$$
$$F^{\mathrm{pol,C6}}_{8,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V^2-g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\Big]$$
$$F^{\mathrm{pol,C6}}_{9,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_V\,g^q_A$$
$$F^{\mathrm{pol,C6}}_{10,q} = e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V^2+g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V$$
$$F^{\mathrm{pol,C6}}_{11,q} = 4\Big[-e_q\,(\mathrm{Re}\,g_Z)\,g^l_A\,g^q_A + |g_Z|^2\,2(g_V g_A)_l\,(g_V g_A)_q\Big]$$
$$F^{\mathrm{pol,C6}}_{12,q} = -e_q^2 - |g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V^2+g_A^2)_q + e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\ ; \tag{26}$$

Case 7:

$$F^{\mathrm{pol,C7}}_{1,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V-g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,(g_V-g_A)_q\Big]$$
$$F^{\mathrm{pol,C7}}_{2,q} = 4\Big[-|g_Z|^2\,2(g_V g_A)_l\,(g_V-g_A)_q^2 + e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_A\,(g_V-g_A)_q\Big]$$
$$F^{\mathrm{pol,C7}}_{3,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_A\,(g_V-g_A)_q$$
$$F^{\mathrm{pol,C7}}_{4,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_A\,g^q_V$$
$$F^{\mathrm{pol,C7}}_{5,q} = e_q\,4\,(\mathrm{Re}\,g_Z)\,g^l_A\,g^q_A$$
$$F^{\mathrm{pol,C7}}_{6,q} = e_q\,8\,(\mathrm{Im}\,g_Z)\,g^l_V\,g^q_A$$
$$F^{\mathrm{pol,C7}}_{7,q} = -4\Big[e_q^2 + |g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V^2-g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\Big]$$
$$F^{\mathrm{pol,C7}}_{8,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V^2-g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\Big]$$
$$F^{\mathrm{pol,C7}}_{9,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_V\,g^q_A$$
$$F^{\mathrm{pol,C7}}_{10,q} = e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V^2+g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V$$
$$F^{\mathrm{pol,C7}}_{11,q} = 4\Big[-e_q\,(\mathrm{Re}\,g_Z)\,g^l_A\,g^q_A + |g_Z|^2\,2\,(g_V g_A)_l\,(g_V g_A)_q\Big]$$
$$F^{\mathrm{pol,C7}}_{12,q} = e_q\,2\,(\mathrm{Im}\,g_Z)\,g^l_A\,g^q_V\ ; \tag{27}$$

Case 8:

$$F^{\mathrm{pol,C8}}_{1,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V-g_A)_q^2 - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,(g_V-g_A)_q\Big]$$
$$F^{\mathrm{pol,C8}}_{2,q} = 8\Big[-|g_Z|^2\,(g_V g_A)_l\,(g_V-g_A)_q^2 + e_q\,(\mathrm{Re}\,g_Z)\,g^l_A\,(g_V-g_A)_q\Big]$$
$$F^{\mathrm{pol,C8}}_{3,q} = -e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_A\,(g_V-g_A)_q$$
$$F^{\mathrm{pol,C8}}_{4,q} = -e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_A\,g^q_V$$
$$F^{\mathrm{pol,C8}}_{5,q} = -e_q\,4\,(\mathrm{Re}\,g_Z)\,g^l_A\,g^q_A$$
$$F^{\mathrm{pol,C8}}_{6,q} = -e_q\,8\,(\mathrm{Im}\,g_Z)\,g^l_V\,g^q_A$$
$$F^{\mathrm{pol,C8}}_{7,q} = 4\Big[e_q^2 + |g_Z|^2\,(g_V^2-g_A^2)_l\,(g_V^2-g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\Big]$$
$$F^{\mathrm{pol,C8}}_{8,q} = 2\Big[e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V^2-g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V\Big]$$
$$F^{\mathrm{pol,C8}}_{9,q} = e_q\,4\,(\mathrm{Im}\,g_Z)\,g^l_V\,g^q_A$$
$$F^{\mathrm{pol,C8}}_{10,q} = e_q^2 + |g_Z|^2\,(g_V^2+g_A^2)_l\,(g_V^2+g_A^2)_q - e_q\,2\,(\mathrm{Re}\,g_Z)\,g^l_V\,g^q_V$$
$$F^{\mathrm{pol,C8}}_{11,q} = 4\Big[-e_q\,(\mathrm{Re}\,g_Z)\,g^l_A\,g^q_A + |g_Z|^2\,2(g_V g_A)_l\,(g_V g_A)_q\Big]$$
$$F^{\mathrm{pol,C8}}_{12,q} = -e_q\,2\,(\mathrm{Im}\,g_Z)\,g^l_A\,g^q_V\ ; \tag{28}$$

Case 9:

$$F^{\mathrm{pol,C9}}_{i,q} = (1/2)\,F^{\mathrm{pol,C4}}_{i,q} \quad (i = 1\text{–}12)\ . \tag{29}$$

We can now compute $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q})$ for any initial-lepton spin state, and at any energy, by using (21)–(29) together with (15) in (17) and (18). We do this first at the Z_0 pole,

$\sqrt{s} = M_z$, where

$$g_z(s = M_z^2) = -\mathrm{i}\,\frac{M_z/\Gamma_z}{4\,\sin^2\theta_W\,\cos^2\theta_W}\,. \tag{30}$$

Taking [8] $\sin^2\theta_W = 0.231$, $M_z = 91.187$ GeV/c^2, and $\Gamma_z = 2.490$ GeV yields, for u-type quarks,

$$\begin{aligned}
&\rho^{\mathrm{pol,C1,C2}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)\\
&\quad = -0.369\,(1+\mathrm{i}\,0.132)\,\frac{\sin^2\theta}{1+\cos^2\theta-1.335\,\cos\theta}\\
&\rho^{\mathrm{pol,C4,C9}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)\\
&\quad = -0.370\,(1-\mathrm{i}\,0.113)\,\frac{\sin^2\theta}{1+\cos^2\theta+1.336\,\cos\theta}\\
&\mathrm{Re}\,[\rho^{\mathrm{pol,C5}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)]\\
&\quad = -0.371\,\frac{0.003-\cos^2\theta}{0.008+\cos^2\theta+0.102\,\cos\theta}\\
&\mathrm{Im}\,[\rho^{\mathrm{pol,C5}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)]\\
&\quad = +0.371\,\frac{0.009+0.047\,\cos\theta}{0.008+\cos^2\theta+0.102\,\cos\theta}\\
&\mathrm{Re}\,[\rho^{\mathrm{pol,C6}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)]\\
&\quad = -0.371\,\frac{1-0.003\,\cos^2\theta}{1+0.008\,\cos^2\theta+0.102\,\cos\theta}\\
&\mathrm{Im}\,[\rho^{\mathrm{pol,C6}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)]\\
&\quad = -0.371\,\frac{0.009\,\cos^2\theta+0.047\,\cos\theta}{1+0.008\,\cos^2\theta+0.102\,\cos\theta}\\
&\mathrm{Re}\,[\rho^{\mathrm{pol,C7}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)]\\
&\quad = -0.374\,\frac{0.911-\cos^2\theta-0.018\,\cos\theta}{1+0.934\,\cos^2\theta+0.195\,\cos\theta}\\
&\mathrm{Im}\,[\rho^{\mathrm{pol,C7}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)]\\
&\quad = +0.374\,\frac{0.009\,\sin^2\theta-1.901\,\cos\theta}{1+0.934\,\cos^2\theta+0.195\,\cos\theta}\\
&\mathrm{Re}\,[\rho^{\mathrm{pol,C8}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)]\\
&\quad = -0.374\,\frac{1-0.911\,\cos^2\theta+0.018\,\cos\theta}{0.934+\cos^2\theta+0.195\,\cos\theta}\\
&\mathrm{Im}\,[\rho^{\mathrm{pol,C8}}_{+-;-+}(u\bar{u};\sqrt{s}=M_z)]\\
&\quad = +0.374\,\frac{0.009\,\sin^2\theta+1.901\,\cos\theta}{0.934+\cos^2\theta+0.195\,\cos\theta}
\end{aligned} \tag{31}$$

and for d-type quarks,

$$\begin{aligned}
&\rho^{\mathrm{pol,C1,C2}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)\\
&\quad = -0.176\,(1+\mathrm{i}\,0.108)\,\frac{\sin^2\theta}{1+\cos^2\theta-1.871\,\cos\theta}\\
&\rho^{\mathrm{pol,C4,C9}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)\\
&\quad = -0.176\,(1-\mathrm{i}\,0.092)\,\frac{\sin^2\theta}{1+\cos^2\theta+1.871\,\cos\theta}\\
&\mathrm{Re}\,[\rho^{\mathrm{pol,C5}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)]\\
&\quad = -0.176\,\frac{0.004-\cos^2\theta}{0.006+\cos^2\theta+0.142\,\cos\theta}\\
&\mathrm{Im}\,[\rho^{\mathrm{pol,C5}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)]\\
&\quad = +0.176\,\frac{0.008+0.069\,\cos\theta}{0.006+\cos^2\theta+0.142\,\cos\theta}\\
&\mathrm{Re}\,[\rho^{\mathrm{pol,C6}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)]\\
&\quad = -0.176\,\frac{1-0.004\,\cos^2\theta}{1+0.006\,\cos^2\theta+0.142\,\cos\theta}\\
&\mathrm{Im}\,[\rho^{\mathrm{pol,C6}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)]\\
&\quad = -0.176\,\frac{0.008\,\cos^2\theta+0.069\,\cos\theta}{1+0.006\,\cos^2\theta+0.142\,\cos\theta}\\
&\mathrm{Re}\,[\rho^{\mathrm{pol,C7}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)]\\
&\quad = -0.184\,\frac{0.872-\cos^2\theta-0.014\,\cos\theta}{1+0.953\,\cos^2\theta+0.276\,\cos\theta}\\
&\mathrm{Im}\,[\rho^{\mathrm{pol,C7}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)]\\
&\quad = +0.184\,\frac{0.007\,\sin^2\theta-1.855\,\cos\theta}{1+0.953\,\cos^2\theta+0.276\,\cos\theta}\\
&\mathrm{Re}\,[\rho^{\mathrm{pol,C8}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)]\\
&\quad = -0.184\,\frac{1-0.872\,\cos^2\theta+0.014\,\cos\theta}{0.953+\cos^2\theta+0.276\,\cos\theta}\\
&\mathrm{Im}\,[\rho^{\mathrm{pol,C8}}_{+-;-+}(d\bar{d};\sqrt{s}=M_z)]\\
&\quad = +0.184\,\frac{0.007\,\sin^2\theta+1.855\,\cos\theta}{0.953+\cos^2\theta+0.276\,\cos\theta}
\end{aligned} \tag{32}$$

At lower energies, at which one can neglect all weak interactions [that is, if $g_z = 0$ in (21)–(29) and quark masses are taken into account] one obtains for any flavor:

$$\begin{aligned}
\rho^{pol,C1,C2,C4,C9}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_z) &= \frac{1}{2}\,\frac{\sin^2\theta}{1+\cos^2\theta+\epsilon^2\sin^2\theta}\\
\rho^{\mathrm{pol,C5}}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_z) &= \frac{1}{2}\\
\rho^{\mathrm{pol,C6}}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_z) &= -\frac{1}{2}\,\frac{\cos^2\theta}{\cos^2\theta+\epsilon^2\sin^2\theta}\\
\mathrm{Re}\,[\rho^{pol,C7,C8}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_z)] &= \frac{1}{2}\,\frac{\sin^2\theta}{1+\cos^2\theta+\epsilon^2\sin^2\theta}\\
\mathrm{Im}\,[\rho^{\mathrm{pol,C7}}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_z)] &= \frac{-\cos\theta}{1+\cos^2\theta+\epsilon^2\sin^2\theta}\\
\mathrm{Im}\,[\rho^{\mathrm{pol,C8}}_{+-;-+}(q\bar{q};\sqrt{s}\ll M_z)] &= \frac{\cos\theta}{1+\cos^2\theta+\epsilon^2\sin^2\theta}\,,
\end{aligned} \tag{33}$$

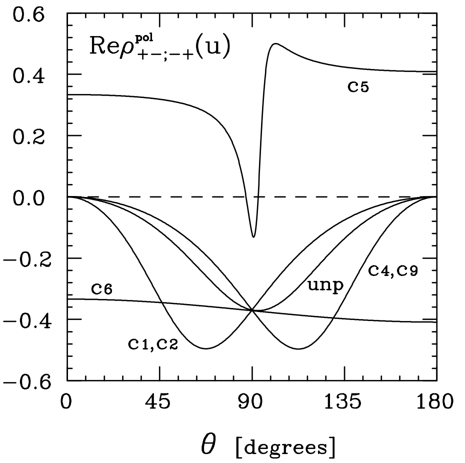

Fig. 1. Plot of $\mathrm{Re}[\rho^{\mathrm{pol}}_{+-;-+}(u\bar{u};\sqrt{s}=M_Z)]$ as a function of θ (the production angle of the vector meson in the e^-e^+ c.m. frame) for cases: C5, C6 (both leptons transversely polarized with spins either parallel or antiparallel); C1, C4 (leptons with opposite helicities); C2, C9 (one lepton longitudinally polarized, the other transversely polarized). The value of $\rho_{+-;-+}(u\bar{u};\sqrt{s}=M_Z)$ for unpolarized leptons is also shown, for comparison. In all other cases, one obtains results similar to the unpolarized case

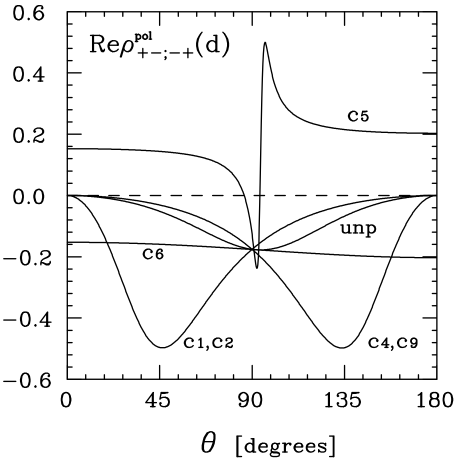

Fig. 2. The same as in Fig. 1, for d-type quarks

where $\epsilon = 2m_q/\sqrt{s}$, which, for heavy flavors, might not be negligible at $\sqrt{s} \ll M_Z$.

Insertion of (31) and (32) or (33) into (1) allows one to make predictions for the relation between $\rho_{1,-1}(V)$ and $\rho_{0,0}(V)$, both of which are measurable quantities. Equation (1) holds for vector mesons that have a large energy fraction x_{E} and are collinear with the parent jet; q is the quark flavor which contributes dominantly to the final vector meson production (e.g., c in D^*); an average should be taken if more than one flavor contributes [3].

Notice that we expect [3] $\rho_{0,0}(V)$ to be independent of the production angle θ, so that the sign of $\rho_{1,-1}(V)$ and its θ dependence are entirely given by the elementary dynamics, via $\rho_{+-;-+}(q\bar{q})$; for unpolarized e^+ and e^-, such dynamics are given by (4) or (5), and for polarized ones, by either (31), (32) or (33). We turn now to a discussion

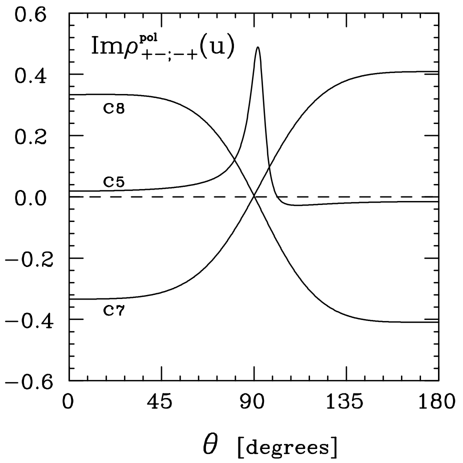

Fig. 3. Plot of $\mathrm{Im}[\rho^{\mathrm{pol}}_{+-;-+}(u\bar{u};\sqrt{s}=M_Z)]$ as a function of θ (the production angle of the vector meson in the e^-e^+ c.m. frame) for cases: C5 (both leptons transversely polarized with spins either parallel or antiparallel); C7, C8 (both leptons transversely polarized, in different directions). In all other cases, *including the unpolarized one*, $\mathrm{Im}[\rho^{\mathrm{pol}}_{+-;-+}(u\bar{u};\sqrt{s}=M_Z)] \simeq 0$

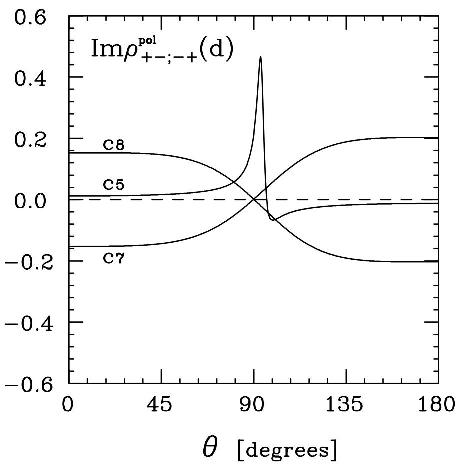

Fig. 4. The same as in Fig. 3, for d-type quarks

of these equations and a comparison with the unpolarized case.

4 Comments and conclusions

We show our numerical results for $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q})$ in Figs. 1–6. We give results only for those cases which strongly differ from the unpolarized case and have such peculiar features that a measurement of $\rho_{1,-1}(V)$ in agreement with them would be an unquestionable test of our approach. In Figs. 1–4, we consider the LEP high-energy case, $\sqrt{s}=M_Z$, and in Figs. 5,6, the lower-energy case, $\sqrt{s} \ll M_Z$.

In Fig. 1, we plot as functions of θ (the V production angle in the e^-e^+ c.m. frame) the real part of $\rho^{\mathrm{pol}}_{+-;-+}(u\bar{u})$ at LEP energy for cases C5, C6, C1,C2, and C4,C9. The value of $\rho_{+-;-+}(u\bar{u})$ for unpolarized leptons is reported

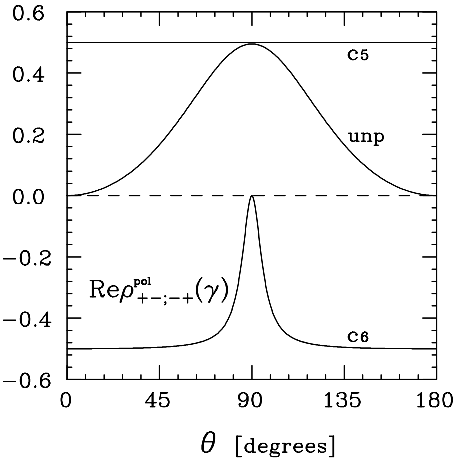

Fig. 5. Plot of $\mathrm{Re}[\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q};\sqrt{s} \ll M_z)]$ as a function of θ (the production angle of the vector meson in the e^-e^+ c.m. frame) for cases C5 and C6 (both leptons transversely polarized with spins either parallel or antiparallel). All other cases give the same result as that given by unpolarized leptons, which is shown for comparison. Quark masses have been taken into account, with $\epsilon = 2m_q/\sqrt{s} = 0.1$

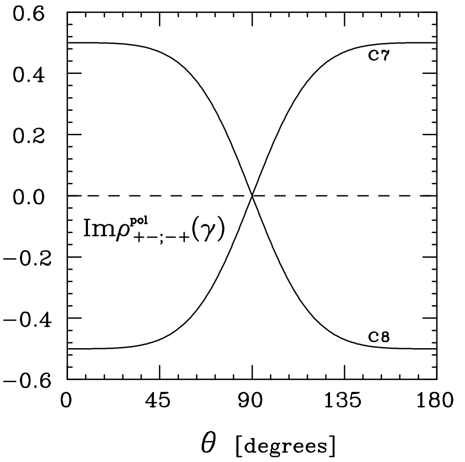

Fig. 6. Plot of $\mathrm{Im}[\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q};\sqrt{s} \ll M_z)]$ for cases C7 and C8 (both leptons transversely polarized, in different directions). In all other cases, *including the unpolarized one*, $\mathrm{Im}[\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q};\sqrt{s} \ll M_z)] = 0$. Again, $\epsilon = 0.1$

also, for comparison. In Fig. 2 we do the same for d-type quarks.

In Fig. 3, we plot the imaginary part of $\rho^{\mathrm{pol}}_{+-;-+}(u\bar{u})$ at LEP energy for cases C5, C7, and C8. In all other cases, *including the unpolarized one*, such an imaginary part is much smaller and should lead to a measurement of Im $\rho_{1,-1}(V) \simeq 0$. The same is done in Fig. 4 for d, s, and b quarks.

In Fig. 5, we plot the real part of $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q};\sqrt{s} \ll M_z)$, taking into account only electromagnetic interactions for cases C5 and C6. All other cases give the same result as that for unpolarized leptons, a result which is reported for comparison. We take quark masses into account, setting $\epsilon = 2m_q/\sqrt{s} = 0.1$.

In Fig. 6, we plot the imaginary part of $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q};\sqrt{s} \ll M_z)$, taking into account only electromagnetic interactions (and quark masses, $\epsilon = 0.1$) for cases C7 and C8. In all other cases, *including the unpolarized one*, the imaginary part is zero.

Figures 1–6 show beyond any possible doubt how the elementary dynamics might lead to very different values of $\rho_{1,-1}(V)$, according to the different spin states of the initial e^+ and e^-. A measurement in agreement with our predictions would confirm in a definite way the necessity of coherent effects in the quark fragmentation and prove all subtleties of the standard model dynamics.

Let us further comment on the most typical cases. The possible spin configurations and the definitions of the various cases are listed at the beginning of Sect. 3. Concerning the real parts at LEP energy – Figs. 1 and 2 – case C5 presents the most striking features, both in sign and θ dependence, and shows a drastic difference from the unpolarized case; also, C6 has a peculiar, almost constant, θ dependence which should be easily detectable. These two cases correspond to e^+ and e^- transversely polarized in the same direction, with either parallel or opposite spins. Cases C1,C2 and C4,C9 also deviate greatly from the unpolarized case, in particular for charge -1/3 quarks: C1 and C4 correspond to initial leptons with opposite helicities and C2, C9 to spin configurations in which one of the leptons is longitudinally polarized and the other is transversely polarized.

Cases C7 and C8, leptons transversely polarized in different directions, lead to results similar to those for unpolarized leptons for the real part of $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q})$; however, in contrast to the unpolarized case, they give large values, strongly varying with θ (see Figs. 3 and 4) for Im $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q})$; this makes them very interesting. Also, C5 exhibits a peculiar θ dependence in Im $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q})$.

At lower energy, when only electromagnetic interactions contribute, cases C5 and C6 are simple and very interesting (see Fig. 5) for the real parts of $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q})$; cases C7 and C8 are unique providers of sizeable imaginary parts of $\rho^{\mathrm{pol}}_{+-;-+}(q\bar{q})$, Fig. 6.

We have thus completed the study of the off-diagonal helicity density matrix element $\rho_{1,-1}(V)$ of vector mesons produced from e^+e^- annihilations into two jets, selecting vector mesons with a large energy fraction (say $x_{\mathrm{E}} \gtrsim 0.5$) and small transverse momentum ($p_{\mathrm{T}}/(x_{\mathrm{E}}\sqrt{s}) \ll 1$) inside one of the jets. The idea was suggested in [1] and [2], and the first numerical predictions, given in [3], have been confirmed by some experimental data [4,5]. We have considered here the most general case of polarized e^+ and e^-; we have given numerical results both at LEP energy, $\sqrt{s} = M_Z$, and for $\sqrt{s} \ll M_z$, but our formulas, (17)–(19) and (21)–(29), are valid at any energy and take into account both electromagnetic and weak interactions.

At the moment, there is no operating e^+e^- collider with polarized beams; however, future generations of linear colliders are being planned, and our study may indicate very good reasons to seriously consider polarization options. Our results have many evident and unambiguous features that cannot be missed by measurements of precision similar to the ones already performed in the unpolarized case [4]–[6], provided that events are carefully selected. The measurement of a sizeable $\rho_{1,-1}$, with its sign, yields immediate valuable and relevant information, allowing direct tests both of the hadronization mechanism and the standard model dynamics.

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